BASIC FACTS: Start with strategies, move on to memorisation

Alex Neill

The educational pendulum has undergone various swings regarding when and whether to memorise basic facts in mathematics. When I was at school, the chanting of basic facts was a feature of my Standard 1 class. Each morning we would recite, "Once two is two, two twos are four, three twos are six" and so on. Some of you may have sung your tables. We were learning our times tables, but did we even know what "times" was? It was, of course, multiplication. I was lucky enough to realise that multiplication was just successively adding on the same number. I sat down and tediously worked out some of the other times tables. Working them out for myself helped me remember them, and I at least had some idea of what they meant. Others may not have been so lucky.

I never did learn my "plus tables". Nobody did back then. Perhaps they still don't. When my Mum was elderly I noticed that her basic facts were good. I once asked her, "9 × 7?" and she instantly said "63". But when I asked her, "What is 9 + 7?" she had to stop and think for some time. She hadn't learnt her "plus tables" either.

Today there has been a tendency to emphasise strategies. This is indeed the key to developing understanding. It can be easy in this environment, however, to marginalise old-fashioned memorisation. There may be a need to re-emphasise the basic facts. What, then, are the basic facts? And why is it important to memorise them?

What are basic facts in mathematics?

Strangely, the answer to this question is that just about anything in mathematics can be a basic fact. In primary school, the main ones encountered are the whole-number basic facts, and in particular addition, multiplication, and their close cousins subtraction and division. As we advance in mathematics we acquire whole bodies of basic facts, such as, "The sum of the angles in a triangle equals 180°", Pythagoras' Theorem, or many other things. Every time we meet one of these we don't try to prove it all over again. We just bring it out of our memory to help solve a problem, or to use it as one step in a more complex line of reasoning. If we didn't do this, we would have to start right back at the beginning for everything we wanted to do in mathematics. A tentative definition of basic facts could be: "Any number or mathematical 'fact' (or idea) that can be instantly recalled without having to resort to a strategy to derive it".

In this paper, I am only going to consider numbers basic facts. Most of the examples in this paper are about whole-number facts, and many relate to the basic addition and multiplication facts. I do, however, want to stress that even these go well beyond just the four arithmetic operators, or the bounds of the 10-times table (e.g., 20 × 6 could easily be recalled). For example, it is a great help to know that one-quarter, 0.25, and 25% are different ways of expressing the same quantity, or that four lots of 25 is 100.

Why learn the basic facts?

The reason is simple: to free the brain up for other aspects of the mathematics we are involved with. If we had to recreate every mathematical piece of information from scratch every time we met it, our brain would soon become overloaded. Scientists have discovered that there is only so much the brain can keep in its working memory. Barrouillet and Fayol (1998) discuss the high cognitive load of employing strategies, even those as simple as counting. Sousa (2008) discusses the three levels of memory: immediate memory (that lasts for seconds); working memory (that lasts for minutes to days); and long-term memory (that lasts for years). He points out that the working memory can only handle a limited number of items, and only for a limited period of time. He states, "Pre-adolescents can handle three to seven items" (p. 51), and that "the time [that items are remembered] is likely to be 5 to 10 minutes" (p. 52). By having facts in automatic long-term memory, working memory can be freed from becoming clogged with more information than it can cope with. This is the role that memorisation plays. It allows facts to be recalled without distracting the brain from the major business in hand—solving the mathematics problem it is faced with.

The other benefit of knowing basic facts is that their acquisition correlates with increased mathematical ability. Pegg, Graham, and Bellert (2005) showed that persistently low-

achieving students made significant gains in their overall mathematics achievement when they had improved automaticity of basic facts. Gray and Mulhern (as cited in Anthony & Knight, 1999) found a significant correlation between the recall of basic facts and mathematical ability, but made no claims of causality.

Start with strategies

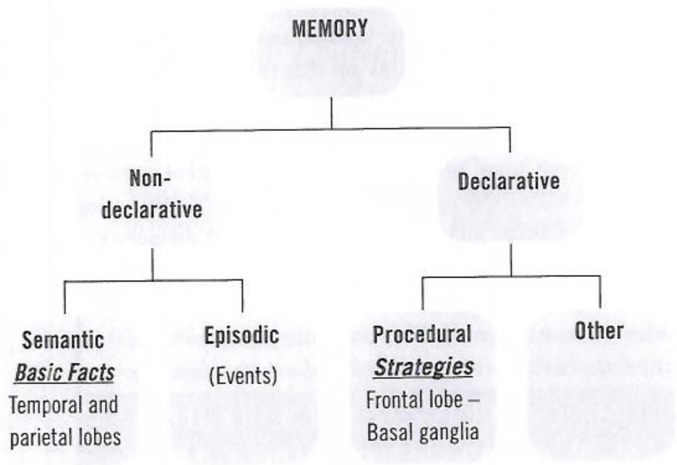
Research has shown that it is inefficient to try to remember facts which do not have a firm meaning to the student. There is little point trying to get a student to learn their times tables if they have no understanding of multiplication. Students should also have a strategy, or preferably a range of strategies, that they can use to obtain results in a particular domain (e.g., multiplication) before we get them memorising facts in that area. As they explore lots of different strategies and contexts, students will start to instantly recall more and more basic facts. If students not only remember which strategies to use and how to perform them, but also understand why the strategy works, they are far more likely to remember basic facts that relate to that strategy. Research has shown that students who develop strategies learn and maintain basic facts more effectively than those who rote learn them (Thinking With Numbers, n.d.).

Baroody (2006) refers to "meaningful memorisation" (p. 25) which is based on the patterns and relationships that underpin the basic facts, and points out that this helps students learn their number facts.

Strategies exist in the so called "non-declarative memory", in particular in the part that pertains to skills and habits. This is often referred to as the procedural part of memory, and this is where the student remembers how to perform a task. Brain scans show that using exact computational strategies activates areas in the left frontal lobe (Dehaene et al., as cited in Sousa, 2008) and in the basal ganglia (Davis, Hill, & Smith, 2000). The memory system is shown schematically in Figure 1.

Explicitly exploring number properties (such as the properties of 0 and 1, or commutativity) is a particularly useful thing to do. Number properties are the building blocks upon which all strategies are actually built. Rather than just using them implicitly, discussing and exploring them explicitly helps build student understanding, and helps them become more algebraic thinkers (Maguire, 2007). Number properties, such as commutativity, allow students to organise the basic facts and make links between them, but also help reduce the number of facts that need to be separately stored. For example, if you know 5 + 2, then 2 + 5 is easily accessible.

FIGURE 1 BASIC FACTS AND MEMORY



Adapted from Davis et al. (2000)

Plenty of practice

Doing a wide variety of work that aims to build and enhance strategies helps reinforce procedures and an understanding of mathematics. Practising strategies also reinforces and continues the process of memorisation. Pinsent and Tait-McCutcheon (n.d.) have an excellent resource on the nzmaths.co.nz website that outlines many strategies and supporting resources. The *Figure It Out* books (Ministry of Education, 1999, 2000, 2001) also have a series on the basic facts which give a mix of strategies and patterns for basic facts, and also practice examples.

Students may find that strategies are so quick that they do not have to memorise the result. This is especially true of addition. But think back to my mother. When asked what 9 + 7 was, she could not automatically recall it. And because her procedural capabilities were slowing down, it became a major effort for her. I suspect that many of us, like her, still revert to strategies for basic addition or subtraction. But remember, using strategies uses up valuable brain capacity that could be employed more usefully.

Move on to memorisation

There is, however, a limit to how far understanding and practising strategies can lead to memorisation. Again, this is related to the way the brain processes information. The automatic recall of facts resides in a different part of the memory system, the so-called "declarative memory" (and in particular in the semantic part of this), whereas strategies reside in the "non-declarative memory" (Figure 1). Brain scans of people who are recalling basic facts show extensive activity in certain specific parts of the brain, in particular the temporal and parietal lobes (Davis et al., 2000). When they are employing strategies, quite different parts of the brain light up. This implies that exploring strategies is effective, but it is not sufficient to ensure full memorisation.

Because different parts of the brain are used for strategies than are used for storing and retrieving facts, many authors believe that separate activities are needed for memorisation than for strategy-based tasks (e.g., Davis et al., 2000). A number of activities for practising basic facts are outlined in *Book 4: Teaching Number Knowledge* (Ministry of Education, 2006a, pp. 32–38).

What is memorisation?

Memorisation is the ability to retrieve facts quickly, accurately, and effortlessly. Three levels of expertise have been identified by Klapp et al. (as cited in Barrouillet & Fayol, 1998):

1. *The novice stage.* Here the student may need to revert to a step-by-step process that they have learnt to obtain the answer. A good example of this is the multiplication tables. To find out what 6 × 3 is, a novice may chant (probably in their heads), "Once three is three, two threes are six, three threes are nine … " until the answer pops out. This is a slow procedure, and it is easy for the novice to be distracted.
2. *The automatised stage.* Here, the repetitive part may still play a role, but the student is both far more rapid with their response and not so easily distracted.
3. *Beyond automaticity.* At this stage the response is even more rapid, and the student will not experience any interference as a result of doing some other task simultaneously. This means that the facts are just there at their fingertips. This is what we really want, because students need to recall the basic fact without it interfering with their focus on the larger mathematical problem. Try calling out a multiplication fact while reciting the alphabet, or counting some objects. If you have no problems, you are beyond automaticity.

Random or sequential access

To achieve automaticity or beyond, students need to have *random access* to basic facts information. By way of example, if the student needs to know the answer to "What is 8 × 4?", then 32 should just pop into mind without their having to go through all or some of the 4-times table sequence (which is sequential access).

Here is a little experiment to see if you have random access to the alphabet or sequential access only:

Experiment 1: What is the 15th letter of the alphabet?

Now, what did you do? Did the correct letter just pop into your mind, and were you very confident that it was the right one? Or did you have to resort to counting (probably on your fingers) while chanting the alphabet to yourself? If you did the latter, then you do not have random access to the alphabet, and are still at the novice stage. Try saying the alphabet backwards and you'll see just how true this is! There is no real point, in fact, in having random access to the alphabet—but random access to number facts is crucial.

Here is another experiment for you:

Experiment 2: What is 8 + 5?

Did the correct number just jump into your head or did you do just a tad of processing? Some of you "just knew". However, some of you may have said something like, "8 + 2 = 10 and 10 + 3 = … ". Some of you may even have counted on (8, 9, 10, 11, …). Again, like my mother, you may not yet be beyond automaticity. I have a hunch that for many of us we would have found that 8 × 5 would have come to mind a little quicker than 8 + 5.

Basic arithmetic facts

For each of the arithmetic operators there are repertoires of strategies that help build up the basic facts. I shall outline some of the key strategies to help progress students' understanding, but stress that strategy activities and memorisation activities should be done separately. A progression that moves from the concrete to the abstract starts with using materials and pictorial representations, moves on through imaging and visualisation, and then moves on to number properties and symbolic representations. Students tend to memorise the basic facts in a similar order to the level of sophistication of the strategy they employ. Remember that strategies ought to be the starting point, with more traditional memorisation methods being employed later. But just doing strategies is not enough, even though it is the best starting point.

Addition

The first and easiest basic facts are the addition ones. Addition can be seen as the joining of two sets. Addition facts include all the additions up to 10 + 9, including adding 0. I have included the "10-plus" facts (e.g., 10 + 6 = 16), as these give us more strategies to get the facts up to 18. Facts that are bigger than 10 + 9 can be deduced from these basic facts.

Research shows that students recall facts with small numbers more easily and quickly than when the numbers are larger. The exception is doubles, which students seem to memorise more easily regardless of the size of the numbers, and groupings to 10 (Barrouillet & Fayol, 1998).

The key facts can be memorised in roughly the order shown below, or in Table 1. The Basic Facts Concept Map in the Assessment Resource Banks (ARBs) (http://arb.nzcer.org.nz/) gives more detailed breakdowns of this:

- Start with additions up to 5, and doubles to 10.
- Learn that adding on 0 leaves the number the same.
- Learn that adding on 1 gives the next counting number.
- Move on to additions up to 10, counting on from the larger number, and doubles to 18.
- Learn groupings to 10, and the effect of adding on from 10.
- Explore additions up to 18, counting on from the larger number.
- Use commutativity to reduce the number of facts to be learnt. Commutativity means that the order in which numbers are added in makes no difference to the final answer, for example, 2 + 5 = 5 + 2. This saves counting on from the smaller number.

Strategies fall into two classes. There are the counting facts. Counting on from the larger number is the more efficient strategy. Secondly there are "derived facts", that is ones which can be deduced from previously known facts. Near doubles, and part–whole partitioning are examples of derived facts.

TABLE 1 ADDITION UP TO 18: COUNTING-ON STRATEGIES

SECOND NUMBER (columns); FIRST NUMBER (rows)

+	0	1	2	3	4	5	6	7	8	9
0										
1						F				D
2			B						D	
3				B				D		
4					B		D			
5	A			E		B			H	
6					D		B			
7				D				B		
8			D			G			B	
9										B
10						C				

KEY (in order of difficulty)

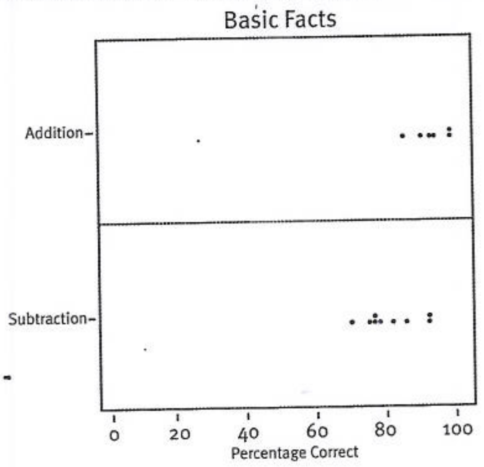
A	Number properties
B	Doubles
C	10-plus
D	Make to 10
E	Count on up to 10
F	Commutativity up to 10
G	Count on through 10
H	Commutativity

Subtraction

Subtraction can be thought of as removing one set of objects from a larger set. This entails *counting back*. It is closely related to the addition facts. For example, if we know 5 + 3 = 8, we can derive the two subtraction facts 8 – 3 = 5 and 8 – 5 = 3.

A study of ARB items showed that students find subtraction harder than addition (Figure 2). Comparing the results from successive National Education Monitoring Project (NEMP) studies showed that 44 percent of Year 4 students got all the 30 basic additions correct (Flockton & Crooks, 1998, p. 14) while only 19 percent of the same students (when in Year 4) got all the subtraction questions correct (Crooks & Flockton, 2002, p. 13). One of the reasons for this is that the commutativity rule does not apply to subtraction, so 5 – 2 ≠ 2 – 5. Crossing the 10 barrier is also harder, and the idea of negative numbers is only a whisker away.

FIGURE 2 DIFFICULTY OF ADDITION AND SUBTRACTION FACTS



(Neill, 2003)

There are two different forms of counting back that can be used in subtraction:

- *Count back by*: What is 9 – 3? *Count back by* 3 gives 9, 8, 7, **6**. This is the most natural form and preserves the fundamental idea of reducing a set by a certain amount. It is not so efficient if the number we are taking away is relatively large. For 9 – 7, we have to count back by 7.
- Count back until: What is 9 – 7? Count back (from 9) until you reach 7—it's **2**. This says, "Count back from the original number until you reach the other number." This strategy is equivalent to using count on from … until … **:** count on from 7 until you reach 9.

TABLE 2 SUBTRACTION BASIC FACTS UP TO 19: ASSOCIATED STRATEGIES

SECOND NUMBER (columns); FIRST NUMBER (rows)

–	0	1	2	3	4	5	6	7	8	9
0										
1		A								
2			A							
3			B	A						
4			C	B	A					
5	A				B	A				
6		B		C		B	A			
7							B	A		
8					C			B	A	
9									B	A
10	A			D		C				B
11		A						E		
12			A				C			
13				A						
14					A			C		
15						A				
16							A		C	
17								A		
18									A	C
19										A

KEY (in order of difficulty)

A	Properties of 0
B	Properties of 1
C	Doubles
D	Count back by
E	Count back until **or** Count on from … until

Multiplication

Multiplication can be thought of as joining together several sets of the same size. The array model in *Book 6: Teaching Multiplication and Division* (Ministry of Education, 2006b, p. 6) is a very useful form of this, especially for the basic facts. Students ought to have moved beyond simple additive thinking into multiplicative thinking before they can usefully acquire the multiplication facts with understanding. We shall look at the facts up to 10 × 10. Strategies for multiplication include repeated addition or skip counting. The commutative property of multiplication almost halves the number of facts that students need to instantly recall.

A suggested order of acquiring the multiplication basic facts is:

Start with the 1×, 2×, 10×, and 5× times tables. These give us 40 basic facts (see the dark, striped columns of Table 3). Commutativity gives us another 24 basic facts; for example, 5 × 7 = 7 × 5 (see the darker, nonstriped rows of Table 3).

Learn the 3× and the 4× times tables. This gives us another 12 basic facts (see the light, striped columns of Table 3). Commutativity gives us another 8 basic facts (see the light, nonstriped rows of Table 3).

Only 16 basic facts remain from the 6×, 7×, 8×, and 9× times tables (see the unshaded parts of Table 3), and six of these can be obtained using commutativity.

TABLE 3 MULTIPLICATION BASIC FACTS TO 100

SECOND NUMBER (columns); FIRST NUMBER (rows)

×	1	2	3	4	5	6	7	8	9	10
1										
2	1×	2×			5×					10×
3			3×	4×						
4	T	T	T	T	T					T
5	A	A			A					A
6	B	B	A	A	B					B
7	L	L	B	B	L					L
8	E	E	L	L	E					E
9			E	E						
10										

Patterns in the times tables

Exploring patterns in the times tables is another useful approach tha both gives interest to and aids the memorisation of the times tables Every table has a repeating pattern of some sort, even the least accessible o all, the 7-times table. Intriguing links can be seen between the last digits ir each number in the times tables. The last digit in each number in a time table (e.g., in the 4-times table) is in the exact reverse order of the last digi of its pair (e.g., in the 6-times table). So, in the 4-times table, the sequenc of last digits is 4, 8, 2, 6, 0, 4, 8, 2, 6, which is reversed in the 6-times tabl (Table 4).

It looks as if this is a feature of having a base-10 number system, an this then explains the commonly known pattern of the 9-times table.

TABLE 4 PATTERNS IN THE ONES DIGIT OF THE TIMES TABLES

Table	Last digit	Table	Last digit
1×	1 2 3 4 5 6 7 8 9	9×	9 18 27 36 45 54 63 72 81
2×	2 4 6 8 10 12 14 16 18	8×	8 16 24 32 40 48 56 64 72
3×	3 6 9 12 15 18 21 24 27	7×	7 14 21 28 35 42 49 56 63
4×	4 8 12 16 20 24 28 32 36	6×	6 12 18 24 30 36 42 48 54

Other multiplication tables

There are multiplication tables that are very useful to know. I will discuss three of them, which are surprisingly easy to learn, and often are already known. They have direct links with basic facts students need in the domain of fractions, decimals, and percentages.

The 11- and the $11.\dot{1}$

The 11-times table and the $11.\dot{1}$-times table are virtually identical. The sequences are simply 11, 22, 33, … , 88, 99; and $11.\dot{1}$, $22.\dot{2}$, $33.\dot{3}$, … , $88.\dot{8}$, 100 (remember, $11.\dot{1}$=11.111111 … going on infinitely often). The $11.\dot{1}$-times table links with the 9-times table, because there are 9 lots of $11.\dot{1}$ in 100. It also links to ninths (1/9), which are written as the decimal $0.\dot{1}$ = 0.1111111 . . .

The 25-times table

Many students already know the doubles fact that 2 × 25 = 50, and that 4 × 25 = 100. Quite a few will also know that 3 × 25 = 75. And that is all you need to know! After 100, the pattern just recycles. The ARB resource NM0163 shows that over 80 percent of Year 10 students knew some of the facts about the 25-times table. This times table links with the 4-times table, because there are 4 lots of 25 in 100. It therefore links to quarters (1/4), which can be written as the decimal 0.25.

The 125-times table

This seems a bit hard, but if we know our 25-times table we are almost there! This is because 2 × 125 = 250, and this looks very like 25. From this we know:

4 × 125 = 500, 6 × 125 = 750, and 8 × 125 = 1000.

So all we need to know is 3 × 125 = 375, 5 × 125 = 625, and 7 × 125 = 875.

This times table links with the 8-times table, because there are 8 lots of 125 in 1000. It therefore links to eighths (1/8), which can be written as the decimal 0.125.

Fractions, decimals, and percentages

Students should move towards the automatic recognition of the relationship between simple fractions, decimals, and percentages (Table 5). I would argue that the order in which we explore these relationships is related to the development of the multiplicative basic facts, and to the way we form decimals. Initially, students deal with decimals to one place, then to two and three places. This is place-value learning. This does, however, relate to multiplication basic facts.

Halves, tenths, and fifths

Let us start with a half (1/2). This is probably the first fraction that we meet. After exploring its fractional form, students explore it as the decimal 0.5, or the percentage 50%. Both of these are closely linked with the 5-times table (50, 100, 150, … **or** 0.5, 1.0, 1.5, 2.0, 2.5, . . .). The next fraction-to-decimal relationship which is fundamental is a tenth (1/10). The decimal form is 0.1, and it is also 10%. This instantly relates to the 10-times table. The third of the trio is a fifth (1/5). This is 0.2 or 20%. And now the 2-times table springs to mind.

Halves, quarters, and eighths

Successive halving is another effective way to look at fractions. Now there are relationships with the 5-, 25-, and 125-times tables that I previously suggested could be learnt. The very important ones are ¼ and ¾, which should be instantly recognised as 0.25 and 0.75, or 25% and 75% respectively.

Ninths, thirds, and sixths

Thirds and ninths are both related to the $11.\dot{1}$-times table, which I have suggested is trivial to learn. Sixths are a little harder, but the only new facts needed are that 1/6 = 16.6% and that 5/6 = 83.3%. This is because 2/6, 3/6, and 4/6 are equivalent to 1/3, 1/2, and 2/3 respectively.

Sevenths

Sevenths are the only unit fractions with nonmemorable percentage or decimal equivalents.

TABLE 5 UNIT FRACTIONS AND THE TIMES TABLES

Fraction	Percentage	Decimal	Related times tables
1/2	50%	0.5	5×
1/10	10%	0.1	10×
1/5	20%	0.2	2×
1/2	50%	0.5	5×
1/4	25%	0.25	25×
1/8	12.5%	0.125	125×
1/9	$11.\dot{1}$%	$0.\dot{1}$	11.1× (or the 11×)
1/3	$33.\dot{3}$%	$0.\dot{3}$	33.3× (or the 11×)
1/6	$16.\dot{6}$%	$0.1\dot{6}$	

When do the basic facts stop?

The simple answer is that they don't. Whatever an individual has in their declarative memory, as opposed to their non-declarative, procedural memory is a basic fact. I have a few peculiar basic facts that have stuck in my mind. For example I know (I'm not sure why) that 7^3 = 343. Maybe this is of marginal use. But I know the square numbers up to 20, and this certainly suggests problem-solving strategies as well as freeing the mind to concentrate on other things. And if I see any of the powers of 2 up to 4096, my mind says "Aha, this could be useful!" And if I see the sequence 1, 3, 6, 10, 15, or 1, 1, 2, 3, 5, 8, 13, a voice inside yells, "Triangular numbers!" or "Fibonacci numbers!" The more of these instant, internal voices that a person has, the more free brain space they will have to solve a problem, and the more clues as to possible methods of solution.

Remember:

- **Start with strategies**
- **Have plenty of practice** with the strategies, then
- **Move on to memorisation** that is automatic, and random.

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